Chapter 3

Problem 10

Please note: there is a factor of "2" different from the book. I do not know where it comes from.

We have for the problem:

```
g[N_, s_] = N! / ((N/2 + s)! (N/2 - s)!)
```

$$\frac{N!}{\left(\frac{N}{2}-s\right)!\,\left(\frac{N}{2}+s\right)!}$$

and

```
2 g[N, 0]
```

$$\frac{2\,N!}{\frac{N}{2}!^{2}}$$

We use the Stirling approximation: $N! \approx (2\pi N)^{1/2} N^N e^{-N}$

```
gapprox[N_, s_] = ((2 π N)^(1/2) N^N e^-N) /
  (((2 π (N/2 + s))^(1/2) (N/2 + s)^(N/2+s) e^(-N/2-s)) ((2 π (N/2 - s))^(1/2) (N/2 - s)^(N/2-s) e^(-N/2+s)))
```

$$\frac{N^{\frac{1}{2}+N}\left(\frac{N}{2}-s\right)^{-\frac{1}{2}-\frac{N}{2}+s}\left(\frac{N}{2}+s\right)^{-\frac{1}{2}-\frac{N}{2}-s}}{\sqrt{2\pi}}$$

```
Log[gapprox[N, 0]]
```

$$\mathrm{Log}\left[\frac{2^{\frac{1}{2}+N}}{\sqrt{N}\,\sqrt{\pi}}\right]$$

```
Log[gapprox[N, s]]
```

$$\text{Log}\left[\frac{N^{\frac{1}{2}+N}\left(\frac{N}{2}-s\right)^{-\frac{1}{2}-\frac{N}{2}+s}\left(\frac{N}{2}+s\right)^{-\frac{1}{2}-\frac{N}{2}-s}}{\sqrt{2\pi}}\right]$$

We expand the powers:

```
Simplify[PowerExpand[Log[gapprox[N, s]]]]
```

$$\frac{1}{2}\left((1+2N)\,\text{Log}[N]+\text{Log}\left[\frac{1}{2\pi}\right]-(1+N-2s)\,\text{Log}\left[\frac{1}{2}(N-2s)\right]-(1+N+2s)\,\text{Log}\left[\frac{N}{2}+s\right]\right)$$

and the logarithms using the Taylor expansion:

```
Series[Log[1 + x], {x, 0, 2}]
```

$$x-\frac{x^2}{2}+O[x]^3$$

And simplify:

$$\text{Simplify}\left[\frac{1}{2}\left((1+2N)\,\text{Log}[N]+\text{Log}\left[\frac{1}{2\pi}\right]-(1+N-2s)\left(\text{Log}\left[\frac{1}{2}N\right]+\left(-\frac{2s}{N}\right)\right)-(1+N+2s)\left(\text{Log}\left[\frac{N}{2}\right]+\left(\frac{2s}{N}\right)\right)\right)\right]$$

$$\frac{-8s^2-N\,\text{Log}[N]+N\left(N\,\text{Log}[4]+\text{Log}\left[\frac{2}{\pi}\right]\right)}{2N}$$

$$-\frac{4s^2}{N}+\text{Log}\left[\frac{2^{N+1/2}}{(\pi N)^{1/2}}\right]$$

$$-\frac{4s^2}{N}+\text{Log}\left[\frac{2^{\frac{1}{2}+N}}{\sqrt{N}\sqrt{\pi}}\right]$$

Problem 7

$$Zb = \frac{1 - e^{-(N+1)\frac{\epsilon}{\tau}}}{1 - e^{-\frac{\epsilon}{\tau}}}$$

$$\frac{1 - e^{\frac{(-1-N)\epsilon}{\tau}}}{1 - e^{-\frac{\epsilon}{\tau}}}$$

The energy is obtained by taking the derivative of the partition function with respect to the temperature:

$$Ub = \tau^2 \partial_\tau \text{Log}[Zb] / \epsilon$$

$$= \frac{e^{\frac{(-N-1)\epsilon}{\tau}} N - e^{\frac{-N\epsilon}{\tau}} N + e^{\frac{(-N-1)\epsilon}{\tau}} - e^{\frac{-\epsilon}{\tau}}}{(-e^{-\epsilon/\tau} + 1)\left(-1 + e^{\frac{(-N-1)\epsilon}{\tau}}\right)}$$

In the case where $\epsilon >> \tau$, then Ub is zero. When the energy levels are separated by an energy much larger than the energy available for that tempreature, the levels will not be occupied.

Problem 6, p. 85, Thermal physics

Partition function,when $\frac{\epsilon}{\tau}$ is small

$$Zs = \text{Integrate}\left[(2x + 1) e^{-x(x+1)\frac{\epsilon}{\tau}}, \{x, 0, \infty\}, \text{Assumptions} \to \left\{\text{Re}\left(\frac{\epsilon}{\tau}\right) > 0\right\}\right]$$

$$\frac{\tau}{\epsilon}$$

Partition function,when $\frac{\epsilon}{\tau}$ is large:

$$Zl = 1 + e^{-2\frac{\epsilon}{\tau}}$$

$$1 + e^{-\frac{2\epsilon}{\tau}}$$

Energy is defined as : $\tau^2 \partial_\tau$ Log[Zs]. Therefore, we obtain :

$$Us = \tau^2 \partial_\tau \text{Log}[Zs]$$

$$\tau$$

Ul = $\tau^2 \partial_\tau$ Log[Zl]

$$\frac{2\, e^{-\frac{2\epsilon}{\tau}}\, \epsilon}{1 + e^{-\frac{2\epsilon}{\tau}}}$$

Heat capacity is defined as : ∂_τ U

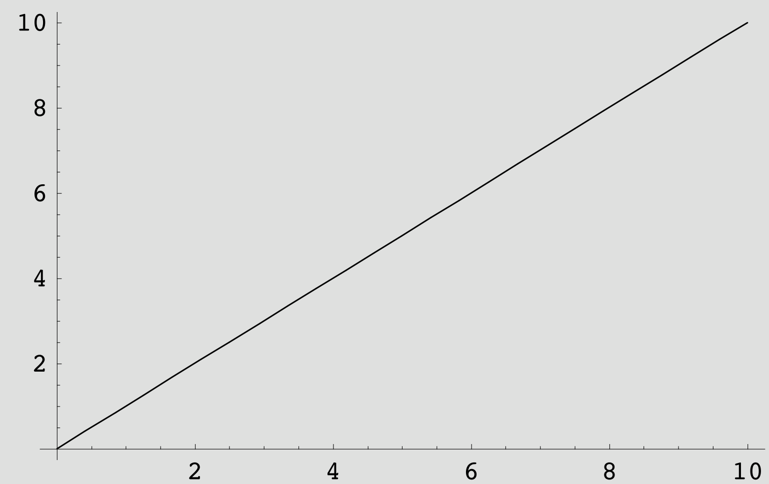
Cs = ∂_τ Us

$$1$$

Cl = ∂_τ Ul

$$\frac{4\, e^{-\frac{2\epsilon}{\tau}}\, \epsilon^2}{\left(1 + e^{-\frac{2\epsilon}{\tau}}\right) \tau^2} - \frac{4\, e^{-\frac{4\epsilon}{\tau}}\, \epsilon^2}{\left(1 + e^{-\frac{2\epsilon}{\tau}}\right)^2 \tau^2}$$

Plot[Us /. {ϵ → 1}, {τ, 0, 10}]
Plot[Cs /. {ϵ → 1}, {τ, 0, 10}]



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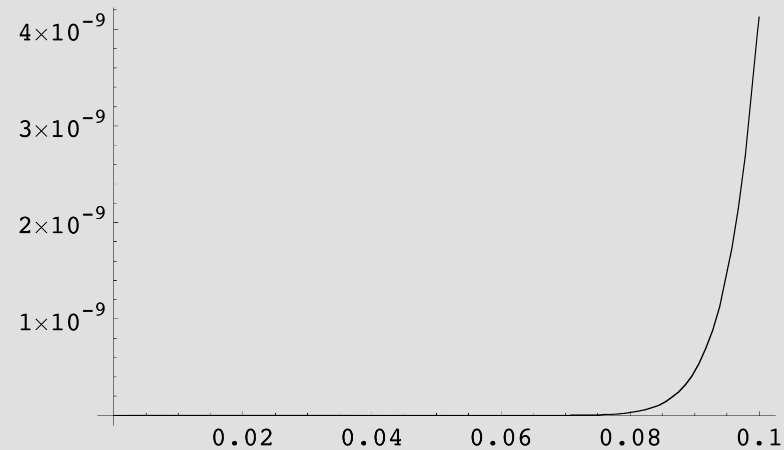
- Graphics -

Plot[Ul /. {ϵ → 1}, {τ, 0, 0.1}, PlotRange → All]
Plot[Cl /. {ϵ → 1}, {τ, 0, 0.1}, PlotRange → All]



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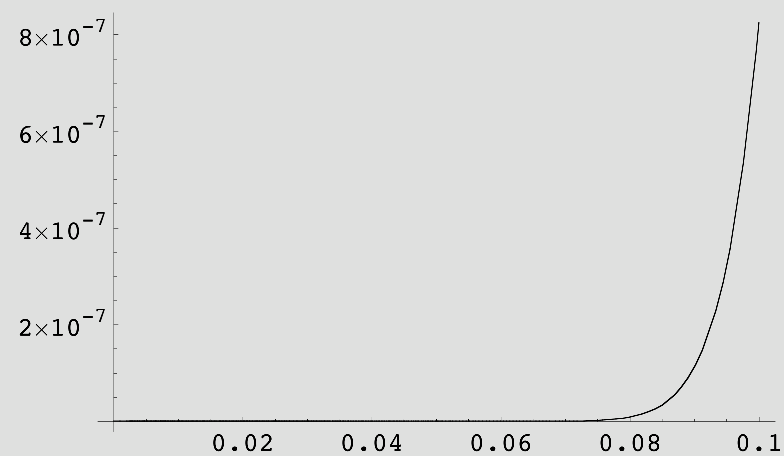

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