Chapter 5

This document presents two problems from Kittel and Kroemer (Thermal Physics).

Note the difference between the treatment of chapter 3 and 5. In chapter 3, we deal with a system in thermal equilibrium with a reservoir (i.e. same temperature for the reservoir and the system). No particles can be exchanged between the reservoir and the system (refer to Fig 3.1, p. 58). In chapter 5, the problem is different: we have two systems in thermal equilibrium with each other as well as with a reservoir (only a single temperature everywhere), and the two systems can exchange energy and particles (refer to figure 5.1, p. 119).

The chemical potential is the change in the Helmholtz energy when changing the number of particles in a system. Look at equation 5.6: if you add one single particle to a system, the Helmholtz energy changes by μ.

Problem 7

We get the partition function:

$$z = \lambda e^{\frac{\Delta}{2\tau}} + e^{\frac{\delta}{2\tau}} + \lambda^2 e^{-\frac{\delta}{2\tau}} + \lambda e^{-\frac{\Delta}{2\tau}}$$

$$e^{\frac{\delta}{2\tau}} + e^{-\frac{\Delta}{2\tau}} \lambda + e^{\frac{\Delta}{2\tau}} \lambda + e^{-\frac{\delta}{2\tau}} \lambda^2$$

with $\lambda = e^{\frac{\mu}{\tau}}$. The number of particles is :

$$\text{NunmberOfParticles} = \frac{\lambda}{z} \partial_\lambda z$$

$$\frac{\lambda \left(e^{-\frac{\Delta}{2\tau}} + e^{\frac{\Delta}{2\tau}} + 2 e^{-\frac{\delta}{2\tau}} \lambda\right)}{e^{\frac{\delta}{2\tau}} + e^{-\frac{\Delta}{2\tau}} \lambda + e^{\frac{\Delta}{2\tau}} \lambda + e^{-\frac{\delta}{2\tau}} \lambda^2}$$

When the number of particles is 1, we have:

$$\lambda \left(e^{-\frac{\Delta}{2\tau}} + e^{\frac{\Delta}{2\tau}} + 2 e^{-\frac{\delta}{2\tau}} \lambda\right) = e^{\frac{\delta}{2\tau}} + e^{-\frac{\Delta}{2\tau}} \lambda + e^{\frac{\Delta}{2\tau}} \lambda + e^{-\frac{\delta}{2\tau}} \lambda^2$$

$$e^{-\frac{\delta}{2\tau}} \left(-e^{\delta/\tau} + \lambda^2\right) == 0$$

Hence, the average number of particles per state will be one in two cases : if $\tau \to 0^+$, then the condition is fulfilled $\left(e^{-\frac{\delta}{2\tau}} \to 0\right)$. The other case leads to $\mu = \delta/2$. We notice that the probability (defined in our case by equation 5.54) is :

$$P\,(N,\,\epsilon) = \frac{e^{(N\,\mu - \epsilon)/\tau}}{z}$$

and therefore, when

$$\mu = \delta / 2$$

the probability of being in the state where N=0 is

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P[0, -δ / 2]
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$$\frac{e^{\frac{\delta}{2\tau}}}{z}$$

and is equal to the probability of being in the state with N=2:

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P[2, δ / 2]
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$$\frac{e^{\frac{\delta}{2\tau}}}{z}$$

Therefore, when $\mu=\delta/2$, the two states where N is 0 or 2 are equally probable and therefore the average occupation is 1.